MAXIMUM A POSTERIORI (MAP) MULTITARGET TRACKING FOR BROADBAND AEROACOUSTIC SOURCES

October 2001

Kristine L. Bell
Dept. of Applied and Engineering Statistics
George Mason University
Fairfax, VA 22030-4444
kbell@gmu.edu

ABSTRACT

Acoustic sensors offer the potential to detect, track, and classify enemy ground targets such as tanks, trucks, and other military vehicles. Traditional multitarget tracking techniques partition the track estimation operation into two isolated processes: direction-of-arrival (DOA) estimation from array snapshot data, followed by track estimation from the DOA estimates. In this paper, a multiple target track estimation method that operates directly from broadband array data is presented. The maximum a-posteriori (MAP) estimator for contact states is derived for temporally uncorrelated broadband target signals and uncorrelated target tracks. This technique extends the MAP multitarget tracking approach developed for narrowband signals in [1]. The track estimator is an iterative algorithm employing a nonlinear programming penalty method in conjunction with an alternating maximization algorithm for obtaining penalized maximum likelihood (PML) DOA estimates. The penalty function couples the DOA estimates from the PML algorithm to the tracker as synthetic measurements, eliminating the data association step of traditional multitarget tracking approaches. It also creates a feedback mechanism to enhance the DOA estimation process. The algorithm is derived as a batch method. A sequential implementation obtained by stepping through the data in short batches is applied to acoustic array time series data from field tests conducted by the Army Research Laboratory.

1. INTRODUCTION

Acoustic sensor arrays offer the potential to detect and track enemy ground targets such as tanks, trucks, and other military vehicles [2]-[7]. The acoustic emissions of ground vehicles are generally broadband in nature and are composed of several discrete harmonically related components and an underlying lower level broadband component. A typical plot of the a target frequency spectrum vs. time as the target moves past the array is shown in Figure 1. Traditional multitarget tracking techniques partition

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
00 Oct 2001	N/A	-

Title and Subtitle Maximum A Posteriori (MAP) Multitarget Tracking for Broadband Aeroacoustic Sources	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) George Mason University Kristine L. Bell Dept. of Applied and Engineering Statistics Fairfax, VA 22030-4444	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) Department of the Army, CECOM RDEC Night Vision & Electronic Sensors Directorate AMSEL-RD-NV-D 10221 Burbeck Road Ft. Belvoir, VA 22060-5806	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)
Distribution/Availability Statement Approved for public release, distribution unlimited	
Supplementary Notes See also ADM201471, Papers from the Meeting of the MSS Specialty Group on Battlefield Acoustic and Seismic Sensing, Magnetic and Electric Field Sensors (2001) Held in Applied Physics Lab, Johns Hopkins Univ, Laurel, MD on 24-26 Oct 2001. Volume 2 (Also includes 1999 and 2000 Meetings), The original document contains color images.	
Abstract	
Subject Terms	
Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU
Number of Pages 15	

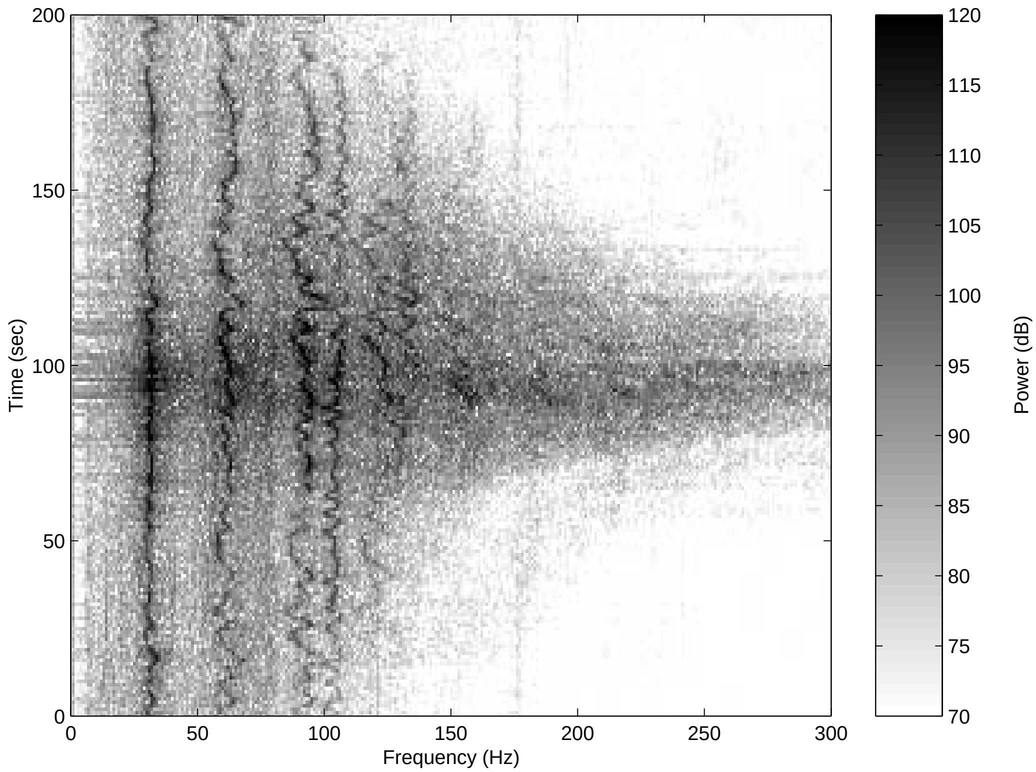

Figure 1: Typical spectrogram for an aeroacoustic ground target.

the track estimation operation into two isolated processes: broadband direction-of-arrival (DOA) estimation from array snapshot data, followed by track estimation from the DOA estimates. This partitioning results in procedures which are suboptimal and require data association to match DOA estimates to targets. In this paper, a multiple target track estimation method that operates directly from broadband array data is presented. The maximum a-posteriori (MAP) estimator for contact states is derived for temporally uncorrelated broadband target signals and uncorrelated target tracks, where the number of targets is assumed known and fixed. This technique extends the MAP multitarget tracking approach developed for narrowband signals in [1]. The track estimator is an iterative algorithm employing a nonlinear programming (NLP) penalty method in conjunction with an alternating maximization algorithm for obtaining penalized maximum likelihood (PML) DOA estimates. The penalty function couples the DOA estimates from the PML algorithm to the tracker as synthetic measurements, eliminating the data association step of traditional multitarget tracking approaches. It also creates a feedback mechanism to enhance the DOA estimation process.

This paper is organized as follows. In Section 2, the signal and motion models are developed for the broadband multitarget tracking problem. The broadband MAP track estimation method is presented in Section 3. In Section 4, a sequential implementation obtained by stepping through the data in short batches is described and tested on acoustic array time series data from field tests conducted at Aberdeen Proving Ground by the Acoustic Signal Processing Branch of the Army Research Laboratory (ARL). Section 5 contains a summary and areas for future research.

2. STATISTICAL MODEL AND ASSUMPTIONS

We consider the multitarget tracking problem where there are M contacts radiating broadband signals received by an array of N sensors. The number of objects M is assumed known and the trajectories of the objects are assumed to be uncorrelated with the trajectories of other objects. The targets and the array are assumed to lie in the $x - y$ plane. The two-dimensional target state is defined as its bearing $u = cos(\theta)$ and bearing rate $\dot{u}$. Thus the state of the mth contact at snapshot k is $\mathbf{x}_{k,m} = [u_{k,m}, \dot{u}_{k,m}]^T$. We assume the motion of the objects is described by a first order Gauss-Markov process, i.e. for the mth contact,

$$\mathbf{x}_{k,m} = \mathbf{F}\mathbf{x}_{k,m-1} + \mathbf{w}_{k,m}, \tag{1}$$

where $\mathbf{F} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$, Δt is the time interval from k to $k+1$, and $\mathbf{w}_{k,m}$ is a zero mean white Gaussian noise process with covariance matrix $\mathbf{Q}$ which is assumed known and fixed over the observation period and equal for all objects. Under these assumptions, the probability density function (pdf) of $\mathbf{x}_{k,m}$ given $\mathbf{x}_{k,m-1}$ is

$$p(\mathbf{x}_{k,m}|\mathbf{x}_{k,m-1}) = \frac{\exp\left\{-\frac{1}{2}(\mathbf{x}_{k,m} - \mathbf{F}\mathbf{x}_{k,m-1})^T\mathbf{Q}^{-1}(\mathbf{x}_{k,m} - \mathbf{F}\mathbf{x}_{k,m-1})\right\}}{2\pi\,|\mathbf{Q}|^{\frac{1}{2}}}, \tag{2}$$

where $|\mathbf{Q}|$ denotes the determinant of $\mathbf{Q}$. There are $\mathcal{K}$ snapshots in an observation batch. No data is available at $k = 0$ so we assume the prior distribution on initial object states is Gaussian with mean $\bar{\mathbf{x}}_{0,m}$ and covariance $\mathbf{\Omega}_{0,m}$,

$$p(\mathbf{x}_{0,m}) = \frac{\exp\left\{-\frac{1}{2}(\mathbf{x}_{0,m} - \bar{\mathbf{x}}_{0,m})^T\mathbf{\Omega}_{0,m}^{-1}(\mathbf{x}_{0,m} - \bar{\mathbf{x}}_{0,m})\right\}}{2\pi\,|\mathbf{\Omega}_{0,m}|^{\frac{1}{2}}}. \tag{3}$$

At the array, we assume that the data has been transformed into the frequency domain and that there are L frequency bins of interest. The $N \times 1$ observed data vector in the lth frequency bin during the kth observation snapshot has the form

$$\mathbf{y}_{k,l} = \sum_{m=1}^{M} s_{k,l,m}\mathbf{v}_l(u_{k,m}) + \mathbf{n}_{k,l}, \tag{4}$$

where $s_{k,l,m}$ is a random signal from the mth source in the lth frequency bin at the kth snapshot with $E[s_{k,l,m}s^*_{k,l,m}] = \alpha_{k,l,m}$. The vector $\mathbf{v}_l(u_{k,m})$ is the $N \times 1$ array response vector for the lth frequency bin to the DOA $u_{k,m}$, and $\mathbf{n}_{k,l}$ is a $N \times 1$ vector of uncorrelated sensor noise samples in the lth frequency bin at the kth snapshot. The source signals and noise are assumed to be sample functions of independent zero-mean Gaussian random processes. It is assumed that the observations are independent from snapshot to snapshot and frequency bin to frequency bin. The signal powers, $\alpha_{k,l,m}$, are assumed to be unknown and to vary across the frequency band and in time. The noise covariance matrix is assumed to be known and vary across frequency with $E[\mathbf{n}_{k,l}\mathbf{n}^H_{k,l}] = \sigma_l^2\mathbf{I}$. Note that the array data depends on the target state $\mathbf{x}_{k,m}$ only through the bearing $u_{k,m}$ and not the bearing rate $\dot{u}_{k,m}$. We will use the notation

$$u_{k,m} = \mathbf{H}\mathbf{x}_{k,m}, \tag{5}$$

with $\mathbf{H} = [1\ 0]$, to denote bearing component of the state in the subsequent derivation.

We denote the collection of target states across all targets at time k as $\mathbf{x}_k \equiv \{\mathbf{x}_{k,1}, \mathbf{x}_{k,2}, \ldots, \mathbf{x}_{k,M}\}$ and the collection of target powers at time k in frequency bin l as $\boldsymbol{\alpha}_{k,l} \equiv \{\alpha_{k,l,1}, \alpha_{k,l,2}, \ldots, \alpha_{k,l,M}\}$. The collection of target powers over targets and frequency at time k is defined as $\boldsymbol{\alpha}_k \equiv \{\alpha_{k,1}, \alpha_{k,2}, \ldots, \alpha_{k,L}\}$, and the collection of data vectors across frequency at time k is defined as $\mathbf{y}_k \equiv \{\mathbf{y}_{k,1}, \mathbf{y}_{k,2}, \ldots, \mathbf{y}_{k,L}\}$. It is also useful to define the collection of target states over the batch (the track) for each target as $\mathbf{X}_m \equiv \{\mathbf{x}_{0,m}, \mathbf{x}_{1,m}, \ldots, \mathbf{x}_{\mathcal{K},m}\}$.

At each snapshot and in each frequency bin, the array data $\mathbf{y}_{k,l}$ is then jointly complex Gaussian with zero mean and covariance matrix

$$\mathbf{K}_{\mathbf{y}_{k,l}}(\mathbf{x}_k, \boldsymbol{\alpha}_{k,l}) = \sum_{m=1}^{M} \alpha_{k,l,m} \mathbf{v}_l(\mathbf{H}\mathbf{x}_{k,m}) \mathbf{v}_l(\mathbf{H}\mathbf{x}_{k,m})^H + \sigma_l^2 \mathbf{I}, \tag{6}$$

and the pdf of the array data conditioned on the target states is given by

$$p(\mathbf{y}_{k,l}|\mathbf{x}_k : \boldsymbol{\alpha}_{k,l}) = \frac{\exp\left\{-\mathbf{y}_{k,l}^H \mathbf{K}_{\mathbf{y}_{k,l}}^{-1}(\mathbf{x}_k, \boldsymbol{\alpha}_{k,l}) \mathbf{y}_{k,l}\right\}}{\pi^N \left|\mathbf{K}_{\mathbf{y}_{k,l}}(\mathbf{x}_k, \boldsymbol{\alpha}_{k,l})\right|}. \tag{7}$$

We have used the notation $p(\mathbf{y}_{k,l}|\mathbf{x}_k : \boldsymbol{\alpha}_{k,l})$ to emphasize that this pdf is conditioned on the random vector $\mathbf{x}_k$ but is also a function of the non-random but unknown parameter vector $\boldsymbol{\alpha}_{k,l}$. At each snapshot the joint pdf of the broadband data conditioned on target states is the product of the pdfs in each frequency bin and is given by:

$$p(\mathbf{y}_k|\mathbf{x}_k : \boldsymbol{\alpha}_k) = \prod_{l=1}^{L} p(\mathbf{y}_{k,l}|\mathbf{x}_k : \boldsymbol{\alpha}_{k,l}). \tag{8}$$

The single snapshot joint pdf of the observations and contact state conditioned on the previous contact state is then

$$p(\mathbf{y}_k, \mathbf{x}_k|\mathbf{x}_{k-1} : \boldsymbol{\alpha}_k) = p(\mathbf{y}_k|\mathbf{x}_k : \boldsymbol{\alpha}_k) \prod_{m=1}^{M} p(\mathbf{x}_{k,m}|\mathbf{x}_{k,m-1}). \tag{9}$$

Let $\mathbf{X}$, $\mathbf{A}$, and $\mathbf{Y}$ denote the batch collections $\mathbf{X} \equiv \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_M\} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{\mathcal{K}}\}$, $\mathbf{A} \equiv \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_{\mathcal{K}}\}$, and $\mathbf{Y} \equiv \{\mathbf{y}_1, \ldots, \mathbf{y}_{\mathcal{K}}\}$. The joint pdf of the array snapshot data and target states over the batch is given by

$$p(\mathbf{Y}, \mathbf{X} : \mathbf{A}) = \prod_{\nu=1}^{M} p(\mathbf{x}_{0,\nu}) \bullet \left(\prod_{k=1}^{\mathcal{K}} p(\mathbf{y}_k|\mathbf{x}_k : \boldsymbol{\alpha}_k) \prod_{m=1}^{M} p(\mathbf{x}_{k,m}|\mathbf{x}_{k,m-1}) \right). \tag{10}$$

3. BROADBAND BATCH MAP ALGORITHM

In the classical single target tracking problem where the observations are a linear function of the target states and the observations and states are Gaussion, the discrete time Kalman filter provides the minimum mean square error (MMSE) and MAP estimates of the target states given the observations. When a batch of observations is used, the MMSE and MAP estimates are obtained from the fixed interval Kalman smoother [8]. In the problem considered here, the observations depend on the target states in a nonlinear

manner, therefore the MMSE and MAP estimates will yield different solutions. We also have the added complication of the unknown nuisance parameter vector $\mathbf{A}$. The MAP methodology provides a tractable framework for solving this problem. We can jointly find the MAP estimate of $\mathbf{X}$ and the maximum likelihood (ML) estimate of $\mathbf{A}$ by maximizing the joint pdf $p(\mathbf{Y}, \mathbf{X} : \mathbf{A})$, or equivalently its logarithm, with respect to both $\mathbf{X}$ and $\mathbf{A}$. In [1] a practical algorithm for batch track estimation for the narrowband problem was developed using the penalty method of nonlinear programming. Earlier versions also appeared in [9] and [10]. In this section, a batch technique for broadband data is presented.

The MAP/ML estimates of $\mathbf{X}$ and $\mathbf{A}$ are the solutions to the optimization problem:

$$\max_{\mathbf{X}, \mathbf{A}} \ \ln \left[\prod_{\nu=1}^{M} p(\mathbf{x}_{0,\nu}) \bullet \left(\prod_{k=1}^{\mathcal{K}} p(\mathbf{y}_k | \mathbf{x}_k : \boldsymbol{\alpha}_k) \prod_{m=1}^{M} p(\mathbf{x}_{k,m} | \mathbf{x}_{k,m-1}) \right) \right]. \tag{11}$$

Following [1], to assist in the solution we introduce a set of auxiliary DOA variables $\mu_{k,m}$ for each target and snapshot, and define collections of these variables as $\boldsymbol{\mu}_k \equiv \{\mu_{k,1}, \mu_{k,2}, \ldots, \mu_{k,M}\}$ and $\mathbf{M} \equiv \{\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_{\mathcal{K}}\}$. We replace $u_{k,m} = \mathbf{H}\mathbf{x}_{k,m}$ with the new auxiliary variable $\mu_{k,m}$ in the $p(\mathbf{y}_k | \mathbf{x}_k : \boldsymbol{\alpha}_k)$ term. In order to retain the original optimization problem, we then require the new variable to be equal to the old variable, i.e. $\mu_{k,m} = u_{k,m} = \mathbf{H}\mathbf{x}_{k,m}$. The unconstrained optimization problem in Eq. (11) can be written as an equivalent constrained optimization problem as follows:

$$\begin{aligned} &\max_{\mathbf{X}, \mathbf{M}, \mathbf{A}} \ \ln \left[\prod_{\nu=1}^{M} p(\mathbf{x}_{0,\nu}) \bullet \left(\prod_{k=1}^{\mathcal{K}} p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) \prod_{m=1}^{M} p(\mathbf{x}_{k,m} | \mathbf{x}_{k,m-1}) \right) \right] \\ &s.t. \quad \mu_{k,m} = u_{k,m} = \mathbf{H}\mathbf{x}_{k,m}, \ \ k = 1, \ldots, \mathcal{K}; m = 1, \ldots, M \end{aligned} \tag{12}$$

where

$$p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) \equiv p(\mathbf{y}_k | \mathbf{x}_k : \boldsymbol{\alpha}_k)|_{\mathbf{H}\mathbf{x}_{k,m} = \mu_{k,m}}. \tag{13}$$

This formulation allows us to use the penalty method of NLP (e.g. [11], [12]) for constrained optimization problems. It is an iterative procedure which involves solving a sequence of easier unconstrained optimization problems. The easier problems are related to the original constrained problem by a continuous, differentiable penalty function which is equal to zero in the feasible region where the constraints are satisfied, and which is negative in the infeasible region. The penalty function relaxes the equality constraint resulting in a problem which is an approximation to the original problem. With each iteration, a stronger penalty is imposed for infeasibility, and the solution to the unconstrained problem converges to the solution to the original constrained problem. An overview of the method and the convergence properties is provided in [1]. As in [1], we choose the quadratic penalty function,

$$P(\mathbf{X}, \mathbf{M}) = -\sum_{k=1}^{\mathcal{K}} \sum_{m=1}^{M} \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2\sigma_{k,m}^2}, \tag{14}$$

where $\sigma_{k,m}^2$ is a parameter that affects the strength of the penalty.

To enforce a more costly penalty at each iteration, a term $(c_q)^{-1}$ scales the penalty function, where q is the iteration index and $c_q, q = 1, 2, \ldots$ is a positive, decreasing sequence converging to zero. The

penalized unconstrained maximization problem is given by

$$\max_{\mathbf{X}, \mathbf{M}, \mathbf{A}} \ \ln\left[\prod_{\nu=1}^{M} p(\mathbf{x}_{0,\nu}) \bullet \left(\prod_{k=1}^{\mathcal{K}} p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) \prod_{m=1}^{M} p(\mathbf{x}_{k,m}|\mathbf{x}_{k,m-1})\right)\right] - \frac{1}{c_q}\sum_{k=1}^{\mathcal{K}}\sum_{m=1}^{M}\frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2\sigma_{k,m}^2}. \tag{15}$$

Expanding and rearranging the terms in Eq. (15), we have

$$\max_{\mathbf{X}, \mathbf{M}, \mathbf{A}} \ \sum_{k=1}^{\mathcal{K}} \ln p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) + \sum_{m=1}^{M} \ln\left[p(\mathbf{x}_{0,m}) \prod_{k=1}^{\mathcal{K}} p(\mathbf{x}_{k,m}|\mathbf{x}_{k,m-1})\right] - \sum_{k=1}^{\mathcal{K}}\sum_{m=1}^{M}\frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2c_q\sigma_{k,m}^2}. \tag{16}$$

Note that the first term is only a function of $\mathbf{M}$ and $\mathbf{A}$, the second term is only a function of $\mathbf{X}$, and the third term provides the coupling between the parameter sets.

First consider that for a fixed $\mathbf{M}$ and $\mathbf{A}$, we can find $\mathbf{X}$ by maximizing over the second and third terms in Eq. (16). These terms decouple with respect to the targets, therefore the problem reduces to solving M separate track estimation problems. Expanding the pdfs, the problem becomes

$$\max_{\mathbf{X}_m} \ \left\{ -(\mathbf{x}_{0,m} - \bar{\mathbf{x}}_{0,m})^T \boldsymbol{\Sigma}_{0,m}^{-1} (\mathbf{x}_{0,m} - \bar{\mathbf{x}}_{0,m}) - \sum_{k=1}^{\mathcal{K}} (\mathbf{x}_{k,m} - \mathbf{F}\mathbf{x}_{k-1,m})^T \mathbf{Q}^{-1} (\mathbf{x}_{k,m} - \mathbf{F}\mathbf{x}_{k-1,m}) - \sum_{k=1}^{\mathcal{K}} \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{c_q\sigma_{k,m}^2} \right\}. \tag{17}$$

This problem has the form of the classical single source tracking problem with $\mu_{k,m}$ acting as the noisy measurements, and $c_q\sigma_{k,m}^2$ the measurement variance. The solution is the fixed interval Kalman smoother [8]. The implementation consists of the standard forward Kalman filter, followed by a backward smoothing filter. The forward Kalman filter is initialized with $\hat{\mathbf{x}}_{0|0} = \bar{\mathbf{x}}_0$ and $\mathbf{P}_{0|0} = \boldsymbol{\Omega}_0$. The state estimates and their error covariance matrices are computed sequentially for $k = 1, \ldots, \mathcal{K}$ using:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1} \tag{18}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^T + \mathbf{Q} \tag{19}$$

$$\mathbf{G}_k = \mathbf{P}_{k|k-1}\mathbf{H}^T\left[\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^T + c_q\sigma_{k,m}^2\right]^{-1} \tag{20}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{G}_k\left[\mu_{k,m} - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}\right] \tag{21}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{G}_k\mathbf{H}\mathbf{P}_{k|k-1}. \tag{22}$$

Once the forward filtering pass has completed, the backward smoothing pass updates the state estimates and covariance matrices for $k = \mathcal{K} - 1, \ldots, 0$ using:

$$\mathbf{B}_k = \mathbf{P}_{k|k}\mathbf{F}^T\mathbf{P}_{k+1|k}^{-1} \tag{23}$$

$$\hat{\mathbf{x}}_{k|\mathcal{K}} = \hat{\mathbf{x}}_{k|k} + \mathbf{B}_k\left[\hat{\mathbf{x}}_{k+1|\mathcal{K}} - \hat{\mathbf{x}}_{k+1|k}\right] \tag{24}$$

$$\mathbf{P}_{k|\mathcal{K}} = \mathbf{P}_{k|k} + \mathbf{B}_k\left[\mathbf{P}_{k+1|\mathcal{K}} - \mathbf{P}_{k+1|k}\right]\mathbf{B}_k^T. \tag{25}$$

The final track estimate for the mth target is $\hat{\mathbf{x}}_{k,m} = \hat{\mathbf{x}}_{k|\mathcal{K}}, k = 1, \ldots, \mathcal{K}$.

Next consider that for a fixed $\mathbf{X}$, we can solve for both $\mathbf{M}$ and $\mathbf{A}$ by maximizing over the first and third terms in Eq. (16). These terms decouple with respect to the snapshots, therefore this problem reduces to solving $\mathcal{K}$ separate multiple source broadband DOA estimation problems of the form

$$\max_{\boldsymbol{\mu}_k, \boldsymbol{\alpha}_k} \ln p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) - \sum_{m=1}^{M} \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2c_q\sigma_{k,m}^2}. \tag{26}$$

This is a maximum penalized likelihood (MPL) estimation problem. In [1], it was solved iteratively using the expectation-maximization (EM) algorithm [13]. Here, we choose to solve it iteratively using the relaxation, or alternating maximization (AM), method because the AM method usually converges faster than the EM algorithm. The derivation is given in Appendix A.

We then alternate between estimating the penalized DOAs and power estimates using the DOA AM algorithm, and the M track estimates via M fixed interval Kalman smoothers. As presented above, there are three levels of iteration: the penalty method iteration in which the penalty parameter forces the solution into the feasible region, the AM iteration between DOA estimation and track estimation, and the AM iteration used in broadband DOA estimation. Each of the iterations may be performed until a convergence criterion is satisfied or for a fixed number of cycles. The trade-off is algorithm complexity versus estimation accuracy. In this paper, we choose to perform the two AM iterations only once for each penalty method iteration, thus there is only one global iteration loop.

The term $c_q\sigma_{k,m}^2$ controls the strength of the penalty in the DOA estimation stage and acts as the measurement error variance in the tracking stage. In [1], it was suggested to set $\sigma_{k,m}^2$ proportional to the Cramer-Rao bound and to let c_q decrease exponentially. Here we take a simpler approach in specifying $\sigma_{k,m}^2$ and set it inversely proportional to the mth target's total estimated power in the kth snapshot, i.e.

$$\sigma_{k,m}^2 = \beta \left(\sum_{l=1}^{L} \hat{\alpha}_{k,l,m} \right)^{-1}. \tag{27}$$

An explicit pseudo-code description of the batch algorithm using this method for setting $\sigma_{k,m}^2$ and the simplified iteration strategy is provided in Table 1.

4. BROADBAND SEQUENTIAL MAP ALGORITHM AND EXPERIMENTAL RESULTS

The batch algorithm provides an elegant solution to multitarget tracking without requiring data association. However, many systems cannot wait while a batch of data is collected and processed, and we would like a real-time (sequential) solution. As suggested in [1], the batch method can be extended to a sequential method by moving through the data with a shorter batch window of length $\mathcal{K}_s$ and a stride of length Δk. In the full batch method $\mathcal{K}_s = \mathcal{K}$ and $\Delta k = 0$, while a fully sequential method would use

Table 1: Batch broadband MAP multitarget tracking algorithm pseudo code.

Initialize $\hat{\mathbf{x}}^0_{k,m}, \hat{\alpha}^0_{k,l,m}, \ \forall k, l, m$

for $q = 1, \ldots, q_{max}$

Bearing and Power Estimates: AM Algorithm

for $k = 1, \ldots, \mathcal{K}$

for $m = 1, \ldots, M$

Update 'Measurement Error' Variance

$$\sigma^2_{k,m} = \beta \left(\sum_{l=1}^{L} \hat{\alpha}^{q-1}_{k,l,m} \right)^{-1}$$

Pre-whiten data, $\forall l$

$$\mathbf{\Sigma}_{k,l,m} = \sum_{m'=1}^{m-1} \hat{\alpha}^q_{k,l,m'} \mathbf{v}_l(\hat{\mu}^q_{k,m'}) \mathbf{v}^H_l(\hat{\mu}^q_{k,m'}) + \sum_{m'=m+1}^{M} \hat{\alpha}^{q-1}_{k,l,m'} \mathbf{v}_l(\hat{\mu}^{q-1}_{k,m'}) \mathbf{v}^H_l(\hat{\mu}^{q-1}_{k,m'}) + \sigma^2_l \mathbf{I}$$

$$\tilde{\mathbf{y}}_{k,l} = \mathbf{\Sigma}^{-1/2}_{k,l,m} \mathbf{y}_{k,l}$$

Bearing estimate

$$\hat{\mu}^q_{k,m} = \underset{\mu}{\operatorname{argmax}} \sum_{l=1}^{L} \left(\frac{\left| \tilde{\mathbf{y}}^H_{k,l} \tilde{\mathbf{v}}_l(\mu) \right|^2 \gamma_l(\mu)}{1 + \gamma_l(\mu) \left| \tilde{\mathbf{v}}_l(\mu) \right|^2} - \ln \left[1 + \gamma_l(\mu) \left| \tilde{\mathbf{v}}_l(\mu) \right|^2 \right] \right) - \frac{\left(\mu - \mathbf{H} \hat{\mathbf{x}}^{q-1}_{k,m} \right)^2}{2 c_q \sigma^2_{k,m}}$$

where $\gamma_l(\mu) = \max \left[\sigma^2_l, \left(\left| \tilde{\mathbf{y}}^H_{k,l} \tilde{\mathbf{v}}_l(\mu) \right|^2 \Big/ \left| \tilde{\mathbf{v}}_l(\mu) \right|^2 - 1 \right) \Big/ \left| \tilde{\mathbf{v}}_l(\mu) \right|^2 \right]$ and $\tilde{\mathbf{v}}_l(\mu) = \mathbf{\Sigma}^{-1/2}_{k,l,m} \mathbf{v}_l(\mu)$

Power Estimates, $\forall l$

$$\hat{\alpha}^q_{k,l,m} = \gamma_l(\hat{\mu}^q_{k,m})$$

end $\{m\}$

end $\{k\}$

Track Estimates: Fixed Interval Kalman Smoother

for $m = 1, \ldots, M$

Initialize $\mathbf{x}_{0|0} \equiv \bar{\mathbf{x}}_{0,m}, \mathbf{P}_{0|0} \equiv \mathbf{\Omega}_{0,m}$

for $k = 1, \ldots, \mathcal{K}$

$$\mathbf{P}_{k|k-1} = \mathbf{F} \mathbf{P}_{k-1|k-1} \mathbf{F}^T + \mathbf{Q}$$

$$\mathbf{G}_k = \mathbf{P}_{k|k-1} \mathbf{H}^T \left\{ \mathbf{H} \mathbf{P}_{k|k-1} \mathbf{H}^T + c_q \sigma^2_{k,m} \right\}^{-1}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{G}_k \mathbf{H} \mathbf{P}_{k|k-1}$$

$$\mathbf{b}_{k|k} = \mathbf{F} \mathbf{b}_{k-1,k-1} + \mathbf{G}_k \left\{ \hat{\mu}^q_{k,m} - \mathbf{H} \mathbf{F} \mathbf{b}_{k-1|k-1} \right\}$$

end $\{k\}$

Set $\hat{\mathbf{x}}^q_{\mathcal{K},m} = \mathbf{b}_{\mathcal{K}|\mathcal{K}}$

for $k = \mathcal{K} - 1, \ldots, 1$

$$\mathbf{B}_k = \mathbf{P}_{k|k} \mathbf{F}^T \mathbf{P}^{-1}_{k+1|k}$$

$$\hat{\mathbf{x}}^q_{k,m} = \mathbf{b}_{k|k} + \mathbf{B}_k [\hat{\mathbf{x}}^q_{k+1,m} - \mathbf{F} \mathbf{b}_{k|k}]$$

$$\hat{\mathbf{P}}^q_{k,m} = \mathbf{P}_{k|k} + \mathbf{B}_k \left[\hat{\mathbf{P}}^q_{k+1,m} - \mathbf{P}_{k+1|k} \right] \mathbf{B}^T_k$$

end $\{k\}$

end $\{m\}$

end $\{q\}$.

$\mathcal{K}_s = 1$ and $\Delta k = 1$. In between, we have batch-sequential methods where Δk is determined by how often state updates are needed, and $\mathcal{K}_s$ is chosen to balance estimation accuracy with algorithmic complexity.

In the ARL data sets, most of the acoustic energy of the targets is concentrated in the frequency band below 200 Hz. The ground vehicles exhibit both broadband energy and strong harmonics which are non-stationary due to vehicle maneuvering and environmental factors. The sensor array is a seven element circular array consisting of six elements in a circle of radius 4 ft., plus one sensor at the center. The array is placed very close to the road traveled by the vehicles, therefore the range and bearing of the targets changes rapidly as the target passes by the sensor. Furthermore, the signal power level varies with range, thus the multiple target scenarios have sources with highly variable power levels as well as fluctuating DOAs. The data is sampled at a frequency of 1024 Hz. The 1024 samples are converted to the frequency domain using a 1024-point FFT to provide adequate frequency resolution, resulting in one frequency domain data snapshot per second. Previous studies have shown that reasonably good DOA estimates can be obtained using the data from 40-85 Hz. Using higher frequency data provides increased resolution at the expense of global estimation errors due to grating lobes in the array beampattern [4],[5]. The large error estimates are extremely detrimental to tracking performance, thus we restrict processing to the lower frequency bins to avoid these errors. The noise power (which is assumed known) was estimated during from a quiet interval in the data when no targets were present.

The data was processed in blocks of $\mathcal{K}_s = 60$ seconds with a stride of $\Delta k = 10$ seconds. The block length was chosen to provide sufficient track estimates for smoothing over periods when a weaker target is masked by a stronger target. The stride was chosen to trade off providing track updates in a timely manner vs. computational complexity. The blocks overlapped by 50 snapshots. State estimates for the overlapping snapshots obtained from the previous block were used as the initial values for the current block. To initialize the state estimates for the new snapshots, we simply projected out in time with the motion model from the most recent state estimate. The batch algorithm was applied to the current block with $q_{max} = 2$ iterations. The final state estimates from this block replaced the estimates from the previous block. To ensure continuity of the final track estimates as the block moved through the data, the prior mean and covariance matrix of the target state for the current block was set equal to the track estimate at the snapshot just prior to the current block, with a zero variance on the DOA and a small variance on the DOA rate. To initialize the algorithm, the tracks in the first block were initialized to have constant DOA and power spectrum equal to the true value at the beginning of the block.

Figures 2 and 3 show results obtained for two scenarios involving two targets. Each figure shows the true DOAs of the targets, the DOA estimates obtained using incoherent minimum variance (IMV) beamforming as in [4], the DOA estimates obtained using broadband ML, and the target tracks from the broadband MAP tracker. In the first scenario, the targets approach from different directions and pass each other close to the array. The targets are widely separated in angle except for a short period of time while they are crossing, and have nearly the same power levels. For this scenario, the single snapshot DOA estimation techniques perform quite well except at the beginning of the experiment and for a short period while the targets are crossing. ML seems to give better estimates than IMV. The tracker is able smooth out the conflicting data and follow both contacts quite well. The tracks deviate a little from the true DOAs during the crossing period, but are able to recover quickly.

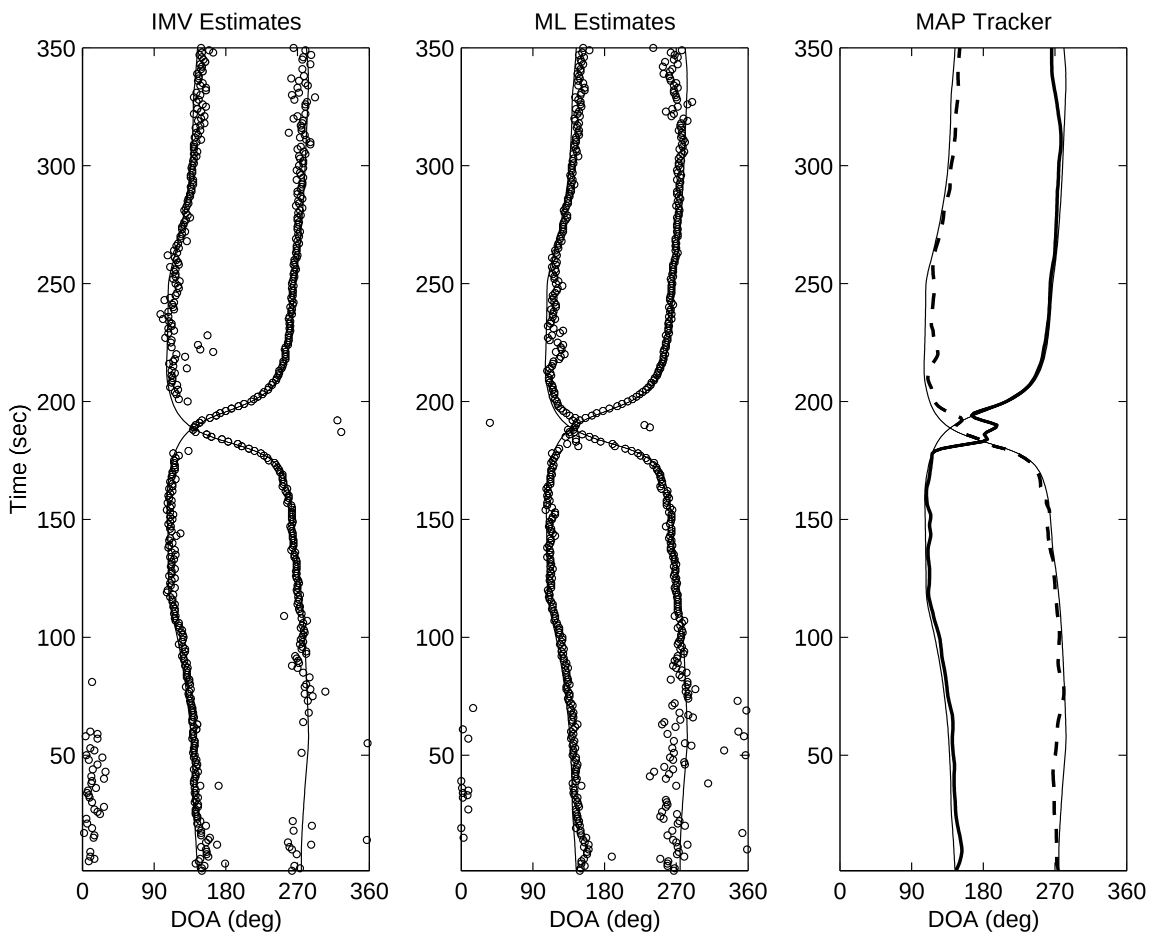

Figure 2: Wideband DOA and MAP tracker estimates for two target crossing scenario.

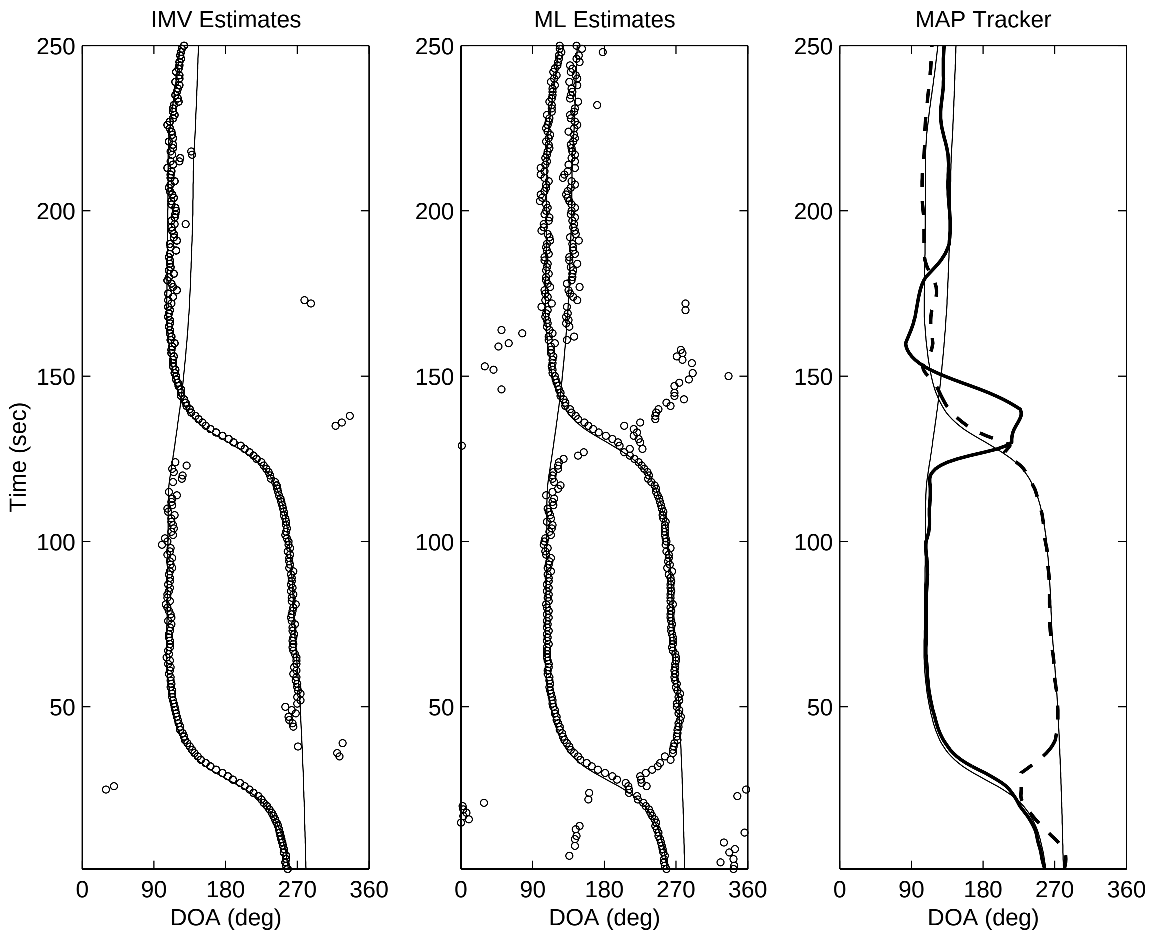

Figure 3: Wideband DOA and MAP tracker estimates for two target following scenario.

In the second scenario, the targets follow one behind another. The targets start out with nearly the same DOA. As the first target passes the array, it's DOA changes rapidly from about 270° to about 90° and it is so loud that it masks the second target. The IMV technique does not report any DOA estimates for the weaker target during this time and the ML technique (which is required to report two estimates) provides random estimates initially, then estimates which are due to a grating lobe from the stronger target. After the first target moves away, the targets are widely separated and both can be heard. Both DOA estimators work well. As the second target approaches the array, it masks the first target. The IMV technique can no longer detect the first target. The ML technique again provides erroneous estimates which can be seen as belonging to energy from a grating lobe of the loud target, then is able to provide reasonably good estimates of the weaker source. This is a difficult scenario for the tracker. Initially, the tracker cannot track the weaker target, but quickly locks on after the first target has passed. The tracker, like the DOA estimators, is able to provide an accurate track of the second target as it moves past the array. The tracker becomes confused when the first target is masked, but is eventually able to recover and track the first contact again. Although the tracker is clearly influenced by the poor ML estimates, the penalty function in the tracker DOA estimator and the smoothing operation allow the tracker to recover.

These results demonstrate that the extra processing performed by the tracker can improve detection and localization of multiple sources over wideband DOA estimation alone. The improvement in performance is due to both the extra smoothing provided by the Kalman smoother, and the focused DOA estimation provided by the MPL formulation.

5. SUMMARY

The narrowband MAP multitarget tracker developed in [1] was extended to handle broadband targets. To construct this algorithm, we introduced an auxiliary DOA parameter and a NLP penalty technique to decouple the Gauss-Markov motion model and the array data model. Convergence of the artificial problem to the original problem was enforced through a stronger penalty with each iteration. In decoupling the problem, DOA and power estimates were obtained from an AM algorithm solution to a maximum penalized likelihood problem, and track estimates were obtained from M single target fixed interval Kalman smoothers, with the DOA estimates serving as noisy measurements. The batch algorithm was implemented in a sequential manner by stepping through the data with short overlapping batches.

Results from acoustic time series data collected by ARL show that the tracker can improve detection and localization of multiple sources over wideband DOA estimation alone. However, the tracker still has trouble during periods when one target is so loud that it masks the other target. One possible solution is to place the sensor array so that it is not so close to roadways or routes taken by the vehicles. This would prevent the array from being overwhelmed by one source, and would decrease the rapidity with which the DOA varies as it moves past the array. Another possibility is to jointly process data from multiple sensor arrays [6],[7]. Not only would this increase the amount of data available, but during periods when a target is close to one array, it would not be close to the rest of the arrays, so target masking will not occur at all arrays simultaneously.

APPENDIX A. BROADBAND ALTERNATING MAXIMIZATION ALGORITHM FOR MPL DOA ESTIMATION

We start from the single snapshot penalized likelihood function in Eq. (26). Expanding terms, it has the form

$$\Lambda \equiv \ln p(\mathbf{y}_k : \boldsymbol{\mu}_k, \boldsymbol{\alpha}_k) = \sum_{l=1}^{L} \ln \left[\frac{\exp\left\{-\mathbf{y}_{k,l}^H \mathbf{K}_{\mathbf{y}_{k,l}}^{-1}(\boldsymbol{\mu}_k, \boldsymbol{\alpha}_{k,l})\mathbf{y}_{k,l}\right\}}{\pi^N \left|\mathbf{K}_{\mathbf{y}_{k,l}}(\boldsymbol{\mu}_k, \boldsymbol{\alpha}_{k,l})\right|} \right] - \sum_{m=1}^{M} \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2c_q\sigma_{k,m}^2}. \quad (28)$$

To reduce notation, we let $\mathbf{K}_{\mathbf{y}_{k,l}} \equiv \mathbf{K}_{\mathbf{y}_{k,l}}(\boldsymbol{\mu}_k, \boldsymbol{\alpha}_{k,l})$ and write (28) as

$$\Lambda = \sum_{l=1}^{L} \left\{-\mathbf{y}_{k,l}^H \mathbf{K}_{\mathbf{y}_{k,l}}^{-1} \mathbf{y}_{k,l}\right\} - \sum_{l=1}^{L} \ln\left\{\pi^N \left|\mathbf{K}_{\mathbf{y}_{k,l}}\right|\right\} - \sum_{m=1}^{M} \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2c_q\sigma_{k,m}^2}. \quad (29)$$

We can use the alternating maximization technique to iteratively solve for parameters associated with one of the M targets by holding all of the parameters associated with the other targets fixed. The likelihood increases at each iteration and convergence to a local maximum is guaranteed. This reduces to a series of MPL problems involving a single source in known *colored* Gaussian noise. Consider estimation of the mth target's DOA $\mu_{k,m}$ and power spectrum $\alpha_{k,1,m}, \ldots, \alpha_{k,L,m}$. We ca rewrite $\mathbf{K}_{\mathbf{y}_{k,l}}$ as

$$\mathbf{K}_{\mathbf{y}_{k,l}} = \alpha_{k,l,m} \mathbf{v}_l(\mu_{k,m}) \mathbf{v}_l(\mu_{k,m})^H + \boldsymbol{\Sigma}_{k,l,m}, \quad (30)$$

where

$$\boldsymbol{\Sigma}_{k,l,m} = \sum_{m' \neq m} \alpha_{k,l,m'} \mathbf{v}_l(\mu_{k,m'}) \mathbf{v}_l^H(\mu_{k,m'}) + \sigma_l^2 \mathbf{I} \quad (31)$$

is the colored noise covariance matrix. It depends only on the noise power and the parameters of the other sources, which are assumed known. Performing a pre-whitening operation on the data, we have

$$\tilde{\mathbf{y}}_{k,l} = \boldsymbol{\Sigma}_{k,l,m}^{-1/2} \mathbf{y}_{k,l}. \quad (32)$$

The pre-whitened data covariance matrix becomes

$$\begin{aligned} \mathbf{K}_{\tilde{\mathbf{y}}_{k,l}} &= \boldsymbol{\Sigma}_{k,l,m}^{-1/2} \mathbf{K}_{\mathbf{y}_{k,l}} \boldsymbol{\Sigma}_{k,l,m}^{-1/2} \\ &= \alpha_{k,l,m} \tilde{\mathbf{v}}_l(\mu_{k,m}) \tilde{\mathbf{v}}_l(\mu_{k,m})^H + \mathbf{I}, \end{aligned} \quad (33)$$

where

$$\tilde{\mathbf{v}}_l(\mu_{k,m}) = \boldsymbol{\Sigma}_{k,l,m}^{-1/2} \mathbf{v}_l(\mu_{k,m}). \quad (34)$$

The penalized log-likelihood function in (29) can be written as

$$\Lambda = \sum_{l=1}^{L} \left\{-\tilde{\mathbf{y}}_{k,l}^H \mathbf{K}_{\tilde{\mathbf{y}}_{k,l}}^{-1} \tilde{\mathbf{y}}_{k,l}\right\} - \sum_{l=1}^{L} \ln\left|\mathbf{K}_{\tilde{\mathbf{y}}_{k,l}}\right| - \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2c_q\sigma_{k,m}^2} + \gamma, \quad (35)$$

where γ is a term which does not depend on the parameters of the mth target. Using the definition in (33), the determinant and inverse of $\mathbf{K}_{\tilde{\mathbf{y}}_{k,l}}$ are given by

$$\left|\mathbf{K}_{\tilde{\mathbf{y}}_{k,l}}\right| = 1 + \alpha_{k,l,m} \left|\tilde{\mathbf{v}}_l(\mu_{k,m})\right|^2 \quad (36)$$

$$\mathbf{K}_{\tilde{\mathbf{y}}_{k,l}}^{-1} = \mathbf{I} - \tilde{\mathbf{v}}_l(\mu_{k,m}) \tilde{\mathbf{v}}_l(\mu_{k,m})^H \frac{\alpha_{k,l,m}}{1 + \alpha_{k,l,m} \left|\tilde{\mathbf{v}}_l(\mu_{k,m})\right|^2}, \quad (37)$$

and the penalized log-likelihood reduces to (dropping unnecessary terms):

$$\Lambda = \sum_{l=1}^{L} \left\{ \left| \tilde{\mathbf{y}}_{k,l}^{H} \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2 \frac{\alpha_{k,l,m}}{1+\alpha_{k,l,m} \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} \right\} - \sum_{l=1}^{L} \ln \left\{ 1+\alpha_{k,l,m} \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2 \right\} - \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2 c_q \sigma_{k,m}^2}. \quad (38)$$

Maximizing over $\alpha_{k,l,m}$ first, we take the derivative and obtain

$$\frac{\partial \Lambda}{\partial \alpha_{k,l,m}} = \frac{\left| \tilde{\mathbf{y}}_{k,l}^{H} \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2}{\left(1+\alpha_{k,l,m} \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2\right)^2} - \frac{\left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2}{1+\alpha_{k,l,m} \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} = 0. \quad (39)$$

Therefore, our estimate is

$$\hat{\alpha}_{k,l,m}(\mu_{k,m}) = \frac{1}{\left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} \left(\frac{\left| \tilde{\mathbf{y}}_{k,l}^{H} \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2}{\left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} - 1 \right). \quad (40)$$

To reduce spurious estimates due to false alarms, we restrict the value of the signal power estimate to be no less than the noise power. Our constrained estimate becomes:

$$\hat{\alpha}_{k,l,m}(\mu_{k,m}) = \max \left[\sigma_l^2, \frac{1}{\left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} \left(\frac{\left| \tilde{\mathbf{y}}_{k,l}^{H} \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2}{\left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} - 1 \right) \right]. \quad (41)$$

To find an estimate of $\mu_{k,m}$, we substitute the estimates $\hat{\alpha}_{k,l,m}(\mu_{k,m})$ for $l = 1, \ldots, L$ back into the penalized log likelihood function in (38) and maximize with respect to $\mu_{k,m}$. There is no closed form expression, so a one-dimensional search is required.

$$\hat{\mu}_{k,m} = \underset{\mu_{k,m}}{\operatorname{argmax}} \sum_{l=1}^{L} \left\{ \left| \tilde{\mathbf{y}}_{k,l}^{H} \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2 \frac{\hat{\alpha}_{k,l,m}(\mu_{k,m})}{1+\hat{\alpha}_{k,l,m}(\mu_{k,m}) \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2} \right\} - \sum_{l=1}^{L} \ln \left\{ 1+\hat{\alpha}_{k,l,m}(\mu_{k,m}) \left| \tilde{\mathbf{v}}_l(\mu_{k,m}) \right|^2 \right\} - \frac{(\mu_{k,m} - \mathbf{H}\mathbf{x}_{k,m})^2}{2 c_q \sigma_{k,m}^2}. \quad (42)$$

The power estimates are then found by substituting the DOA estimate $\hat{\mu}_{k,m}$ back into (41) for $l = 1, \ldots, L$.

ACKNOWLEDGEMENT

The research reported in this document was performed in connection with contract number DAAD-17-00-P-1426 with the U.S. Army Research Laboratory. The views and conclusions contained herein are those of the author and should not be interpreted as presenting the official policies or position, either expressed or implied, of the U.S. Army Research Laboratory or the U.S. Government unless so designated by other authorized documents. Citation of a manufacturer's or trade names does not constitute an official endorsement or approval of the use thereof. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

REFERENCES

1. R. E. Zarnich, K. L. Bell, and H. L. Van Trees, ”A Unified Method for Measurement and Tracking of Multiple Contacts from Sensor Array Data,” *IEEE Trans. Signal Processing*, to appear Dec. 2001.

2. T. Pham and B. M. Sadler, “Wideband Acoustic Array Processing to Detect and Track Ground Vehicles,” *ARL Sensors and Electron Devices Symposium*, (College Park, MD), January 1997.

3. T. Pham and B. M. Sadler, “Focused Wideband Array Processing Algorithms for High-Resolution Direction Finding,” *1998 Meeting of the IRIS Specialty Group on Acoustic and Seismic Sensing*, September 1998.

4. K. L. Bell, “Separation of Multiple Battlefield Acoustic Targets Using Wideband DF-based Adaptive Beamforming, *1999 Meeting of the IRIS Specialty Group on Battlefield Acoustics and Seismics*, September 1999.

5. J. S. Bergin and K. L. Bell, “Wideband Direction-of-Arrival (DOA) Estimation for Multiple Aeroacoustic Sources,” *2000 Meeting of the MSS Specialty Group on Battlefield Acoustics and Seismics*, October 2000.

6. R. J. Kozick and B. M. Sadler, “Algorithms for Localization and Tracking of Acoustic Sources with Widely Separated Sensors,” *2000 Meeting of the MSS Specialty Group on Battlefield Acoustics and Seismics*, October 2000.

7. B. T. Mays and J. Price, “A Multi-Tier Cluster Based Tracker Approach for Battlefield Acoustic Systems,” *2000 Meeting of the MSS Specialty Group on Battlefield Acoustics and Seismics*, October 2000.

8. Y. Bar-Shalom and X. R. Li, *Estimation and Tracking: Principles, Techniques and Software*. Storrs, CT: YBS Publishing, 1998.

9. R. E. Zarnich, K. L. Bell, and H. L. Van Trees, “A Unified Method for Passive Measurement and Tracking of Contacts from an Array of Sensors”, *Proceedings of 8th Annual Adaptive Sensor Array Processing Workshop*, MIT Lincoln Lab, Lexington, MA, March 2000.

10. R. E. Zarnich, K. L. Bell, and H. L. Van Trees, “A Sequential Extension of the Unified MAP Track Estimation Method,” *Proceedings of 1st IEEE Sensor Array and Multichannel Signal Processing Workshop*, Cambridge, MA, March 2000.

11. S. G. Nash and A. Sofer, *Linear and Nonlinear Programming*. New York, NY: McGraw-Hill, 1996.

12. W. I. Zangwill, *Nonlinear Programming: A Unified Approach*. Englewood Cliffs, NJ: Prentice-Hall, 1969.

13. P. J. Green, “On Use of the EM Algorithm for Penalized Likelihood Estimation,” *Journal of the Royal Statistical Society, Series B*, vol. 52, no. 3, pp. 443-452, 1990.